CUET Mathematics Solved Paper-2022

Held on 22 August 2022

1. If the matrices $\begin{bmatrix} 1-x^2 & 2y \\ x+5 & 0 \end{bmatrix} = \begin{bmatrix} -3 & y-1 \\ 7 & 0 \end{bmatrix}$ then the value of x and y is equal to:
   (a) $x=2, y=-1$ (b) $x=-2, y=-1$
   (c) $x=2, y=-2$ (d) $x=2, -2, y=-1$

2. Match List-I with List-II

List-I		List-II	
A.	If $A' = -A$, then matrix A is	I.	Symmetric
B.	If $A' = A$, then matrix A is	II.	$(A')^{-1}$
C.	$(A^{-1})' =$	III.	B'A'
D.	$(AB)' =$	IV.	Skew-symmetric

Choose the correct answer from the option given below:
   (a) A–IV, B–I, C–III, D–II
   (b) A–IV, B–I, C–II, D–III
   (c) A–I, B–IV, C–II, D–III
   (d) A–I, B–IV, C–III, D–II

3. For what value of k the following system of linear equations is:
   $kx+3y+3z=5;\ x-2y+z=-4$
   $3x-y-2z=3$
   has no solution?
   (a) $k=2$ (b) $k=6$
   (c) $k=-3$ (d) $k=-6$

4. If $e^y(x+1)=1$ and $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^k$ then the least positive integer 'k' is equal to
   (a) 1 (b) 2 (c) 3 (d) 4

5. The tangent to the curve $y=e^{2x}$ at the point (0, 1) meets x-axis at:
   (a) (0, 1) (b) $\left(-\frac{1}{2}, 0\right)$
   (c) (2, 0) (d) (0, 2)

6. The interval on which the function $f(x) = -2x^3 - 9x^2 - 12x + 1$ is inereasing, is:
   (a) $(-2, -1)$
   (b) $(-1, \infty)$
   (c) $(-\infty, -2) \cup (-1, \infty)$
   (d) $(-\infty, -2)$

7. If $\int \frac{x}{\sqrt{x-1}} dx = f(x)$ and solution of this integral is $f(x) = A(x-1)^{3/2} + B\sqrt{x-1} + C$, then which of the followings are true?
   (A) $A=\frac{3}{2}, B=\frac{1}{2}$ (B) $A=\frac{2}{3}, B=2$
   (C) $A=\frac{2}{3}, B=-2$ (D) If $f(2)=0$, then $C=\frac{8}{3}$
   (E) If $f(1)=0$, then $C=1$
   Choose the correct answer from the option given below:
   (a) A, D only (b) B, E only
   (c) B, D only (d) D, E only

8. $\int_0^5 |x-5|\,dx$
   (a) $\frac{25}{2}$ (b) 25 (c) 26 (d) $\frac{26}{2}$

9. The area of the region bounded by the curve $y=|x-1|$ and line $x=0$ and $x=2$ in first quadrant is
   (a) 2 (b) 0 (c) 3 (d) 1

10. Match List-I with List-II.

List-I Differential equation	List-II Degree of differential equation
A. $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = \left(\frac{dy}{dx}\right)$	I. 1
B. $\left(1+\frac{dy}{dx}\right)^3 = \left(\frac{d^2y}{dx^2}\right)^3$	II. 2
C. $\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2 = 0$	III. 4
D. $\left(\frac{d^2y}{dx^2}\right)^4 + 2 = \left(\frac{dy}{dx}\right)^2$	IV. 3

Choose the correct answer from the option given below:
   (a) A-I, B-II, C-IV, D-III
   (b) A-IV, B-III, C-I, D-II
   (c) A-II, B-IV, C-I, D-III
   (d) A-III, B-I, C-II, D-IV

11. The differential equation representing family of curve $y = ae^{mx} + be^{nx}$ is
   (a) $y'' - (m+n)y' + mny = 0$
   (b) $y'' + (m+n)y' + mny = 0$
   (c) $y'' - (m+n)y' - mny = 0$
   (d) $y'' - (m+n)y' + y = 0$

12. Two dices are thrown simultaneously. If X denotes the number of sixes, then the expectation of X is
   (a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{8}$

13. In a binomial distribution b(n, p), if $n=10$ and $q=0.7$, then its mean is
   (a) 7 (b) 5 (c) 3 (d) 1

14. Arrange the steps of solving linear programming problem of two variables x and y is referred as "CORNER POINT METHOD" for a bounded region

A. Evaluate the objective function $z = ax + by$ at each corner point.

B. The corner points is determined by solving the equations of the lines intersecting at that point.

C. Find the feasible region by drawing graph using constraints of LPP.

D. When the feasible region is bounded M and m are the maximum and minimum values of z.

E. Let M and m respectively denote the largest and smallest values of evaluating from $z = ax + by$ from all of the corner points.

(a) A–B–D–E–C (b) A–C–D–B–E
(c) C–B–A–E–D (d) E–A–B–C–D

15. The corner points of the feasible region, determined by the system of linear inequalities are (0, 0) (5, 0) (3, 4) and (0, 5). Let $z = 2x + qy$ where $q > 0$. For what value of q the maximum value of z occurs at both (3, 4) and (0, 5) is

(a) 2 (b) 1 (c) $\frac{2}{3}$ (d) 6

16. Let R be the relation in the set {1, 2, 3, 4} and given by R = {(1,2), (2,2), (1, 1), (1,3), (4, 4), (3,3), (3, 2)}. Choose the correct choice

(a) R is reflexive and symmetric, but not transitive
(b) R is reflexive and transitive, but not symmetric
(c) R is symmetric and transitive, but not reflexive
(d) R is an equivalence relation

17. Which one of the following functions is one-one?

A. $f: R \to R$ such that $f(x) = x^2$
B. $f: R \to R$ such that $f(x) = 3x + 2$
C. $f: N \to N$ such that $f(x) = x^2$
D. $f: R \to R$ such that $f(x) = [x]$, where [x] is greatest integer $\leq x$

(a) A and C only (b) B and C only
(c) C and D only (d) A, B and C only

18. The value of $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$ is

(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

19. The value of $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ is:

(a) independent of θ
(b) dependent of θ
(c) equal to x^3
(d) depend on both θ and x

20. One branch of $\sin^{-1}x$ other than the principal value branch corresponds to:

(a) $[\pi, 2\pi]$ (b) $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$
(c) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$ (d) $\left[\frac{3\pi}{2}, \frac{5\pi}{2}\right]$

21. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ where I is the identity matrix of order 2 × 2 and O is 2 × 2 zero matrix, then choose one of the following correct option given below:

(a) $A^2 - 4A + I = 0$ (b) $A^2 + 4A + I = 0$
(c) $A^2 - 4A - I = 0$ (d) $A^2 - 4A + 2I = 0$

22. The function $f(x) = [x]$, where [x] denotes the greatest integer function, is continuous at

(a) 2 (b) 0 (c) –2 (d) 2.5

23. The points of discontinuity for the given function $f: R \to R$ defined as

$$f(x) = \begin{cases} 2x - 5 & 2 < x < 3 \\ 2, & 3 \leq x < 5 \\ 5x, & 5 \leq x < 7 \end{cases}$$

(A) 2 (B) 3 (C) 5 (D) 7

(a) A and B only (b) B and C only
(c) C and D only (d) A and D only

24. The function $f(x) = |x - 1| + |x - 2|$ is not differentiable at

(A) 1 (B) 2 (C) $\frac{3}{2}$ (D) 0
(E) 4

(a) A and B only (b) A and C only
(c) C and D only (d) D and E only

25. A window is in the form of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 10m. Find the side of triangle to admit maximum light. Write the steps to solve this problem in correct order.

A. Assume side of triangle as x and other side of rectangle as y

B. Write perimeter: $3x + 2y = 10$ and area $(A) = xy + \frac{\sqrt{3}}{4}x^2$

C. Put $\frac{dA}{dx} = 0$ to get value of x

D. Write area (A) in terms of x and find $\frac{dA}{dx}$

E. Check $\frac{d^2A}{dx^2} < 0$ at value of x. Then x is required side of triangle.

Choose the correct order from the option given below:

(a) A, B, D, C, E (b) B, A, C, D, E
(c) B, D, A, C, E (d) A, B, C, D, E

26. $\int e^{2x} \sin x \, dx =$

(a) $\frac{e^{2x}}{4}(2\sin x - \cos x) + C$

(b) $\frac{e^{2x}}{5}(2\cos x - \sin x) + C$

(c) $\frac{e^{2x}}{4}(2\sin x + \cos x) + C$

(d) $\frac{e^{2x}}{5}(2\sin x - \cos x) + C$

27. Match List-I with List-II

List-I	List-II
A. $\int \frac{1}{\sqrt{a^2-x^2}}dx$	I. $\cos^{-1}\left(\frac{x}{a}\right)+C$
B. $\int \frac{-1}{(a^2+x^2)}dx$	II. $\frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right)+C$
C. $\int \frac{1}{x\sqrt{x^2-a^2}}dx$	III. $\frac{1}{a}\cot^{-1}\left(\frac{x}{a}\right)+C$
D. $\int \frac{-1}{\sqrt{a^2-x^2}}dx$	IV. $\sin^{-1}\left(\frac{x}{a}\right)+C$

Choose the correct answer from the option given below:
(a) A–I, B–II, C–III, D-IV
(b) A–I, B–II, C–IV, D-III
(c) A–III, B–IV, C–II, D-I
(d) A–IV, B–III, C–II, D-I

28. $\int_0^{\frac{\pi}{2}} \frac{dx}{1+\tan^n x} =$

(a) $\frac{\pi}{2}$ (b) $-\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $-\frac{\pi}{4}$

29. Write the values of the following definite integration in increasing order

A. $\int_0^{\frac{\pi}{2}} \sin^5 x\,dx$

B. $\int_0^{\frac{\pi}{2}} \log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$

C. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+\sqrt{\tan x}}dx$ D. $\int_0^1 xe^x dx$

Choose the correct answer from the option given below:
(a) B<D<A<C (b) D<B<C<A
(c) B<A<C<D (d) B<C<A<D

30. $\int \frac{\sin(x+\alpha)}{\sin x}dx =$
(a) $x\cos\alpha + \sin\alpha \log(\sin x)+C$
(b) $x\cos\alpha - \sin\alpha \log(\sin x)+C$
(c) $x\sin\alpha + \cos\alpha \log(\sin x)+C$
(d) $x\sin\alpha - \cos\alpha \log(\cos x)+C$

31. Arrange the value of all integrals in its ascending order

A. $\int_0^{\frac{\pi}{2}} \frac{\sin^4 x}{\sin^4 x+\cos^4 x}dx$

B. $\int_{-1}^{1} \sin^5 x\cos^4 x\,dx$

C. $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$ D. $\int_2^3 x^2 dx$

Choose the correct option answer from the option given below:
(a) A<B<C<D (b) B<C<A<D
(c) C<A<B<D (d) D<C<B<A

32. If $\int_0^a \frac{\cos x}{1+\sin^2 x}dx = \frac{\pi}{4}$, then the value of a is

(a) 1 (c) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$

33. The solution of differential equation $xy\,dy=(x^2+y^2)dx$ is
(a) $y^2=x^2\log(Cx)$ (b) $y=x^2\log x$
(c) $y^2=x\log(Cx)$ (d) $y^2=2x^2\log(Cx)$

34. Match List-I with List-II

List-I	List-II
A. The distance of the point (3, 4, 5) from y-axis is	I. $\cos^{-1}\left(\frac{1}{3}\right)$
B. Angle between two diagonals of a cube is	II. $\sqrt{34}$
C. Projection of $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ on vector $\vec{b}=2\hat{i}-\hat{j}+\hat{k}$ is	III. $\cos^{-1}\left(-\frac{1}{3}\right)$
D. Angle between $\vec{a}=-\hat{i}-\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}+\hat{k}$ is	IV. $\frac{2}{\sqrt{6}}$

Choose the correct answer from the option given below:
(a) A – II, B – III, C – IV, D – I
(b) A – IV, B – I, C – III, D – II
(c) A – II, B – I, C – III, D – IV
(d) A – II, B – I, C – IV, D – III

35. The shortest distance of the point (3, –2, 1) from the plane $x+2y-2z+9=0$ is
(a) $\frac{1}{2}$ (b) 12 (c) 2 (d) –2

36. Arrange the distance between the following planes to the plane $x+y-z+2=0$ in ascending order:
A. $x+y-z+5=0$
B. $2x+2y-2z+5=0$
C. $3x+3y-3z+9=0$
D. $\frac{1}{2}x+\frac{1}{2}y-\frac{1}{2}z+1=0$

Choose the correct answer from the options given below:
(a) A, B, C, D (b) D, B, C, A
(c) B, A, D, C (d) D, B, A, C

37. If a line $\frac{x-4}{2}=\frac{y}{3}=\frac{z+2}{4}$ intersects a plane $2x-2y+z=0$ at point P, then the distance from point P to the origin is:
(a) 3 units (b) 5 units
(c) 7 units (d) $2\sqrt{2}$ units

38. If A and B are independent events such that $P(A)=\frac{1}{4}$ and $P(B)=\frac{1}{2}$, then $P(A'/B')$ is

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{8}$

39. A random variable has the following probability distribution:

x	0	1	2	3	4
P(X=x)	k	$\frac{1}{4}$	6k	4k	k

Which of the following statements are correct?

A. $k=\frac{1}{10}$

B. $P(X=3)=\frac{1}{4}$

C. $P(X<2)=\frac{11}{16}$

D. $P(1<X<4)=\frac{5}{8}$

E. $P(X=4)=\frac{1}{8}$

(a) A, B, E only (b) B, C, D only

(c) B, D only (d) D, E only

40. In a linear programming problem, Maximize $z=3x+9y$, the corner points are (0, 10), (5, 5), (15, 15) and (0, 20), z is maximum at

A. (0, 10)

B. (5, 5)

C. (15, 15)

D. (0, 20)

(a) A and B only (b) A and C only

(c) B and C only (d) C and D only

Passage

Rahul went to a park with his younger brother and observed that park is circular and a pavement running around it. He asked the gardener about the radius of the park and found it to be 15m. And the pavement is 2m broad. Answer the following questions based on the above information.

41. The area of the circular park, using integration, is given by

(a) $\int_0^{15}\sqrt{225-x^2}\,dx$ (b) $\int_0^{15}\sqrt{15-x^2}\,dx$

(c) $4\int_0^{15}\sqrt{225-x^2}\,dx$ (d) $4\int_0^{15}\sqrt{15-x^2}\,dx$

42. The integral $\sqrt{225-x^2}$ can be solved using the formula

(a) $\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+C$

(b) $\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|+C$

(c) $\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C$

(d) $\frac{x}{2}\sqrt{a^2-x^2}-\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C$

43. The area of the pavement running around the circular park, using integration, is given by

(a) $4\int_0^{17}\sqrt{17-x^2}\,dx$

(b) $4\int_0^{17}\sqrt{289-x^2}\,dx$

(c) $\left(4\int_0^{17}\sqrt{17-x^2}\,dx\right)-\left(4\int_0^{15}\sqrt{15-x^2}\,dx\right)$

(d) $\left(4\int_0^{17}\sqrt{289-x^2}\,dx\right)-\left(4\int_0^{15}\sqrt{225-x^2}\,dx\right)$

44. The area of the circular pavement is

(a) 289π sq units (b) 225π sq units

(c) 514π sq units (d) 64π sq units

45. If the gardener wishes to put manure at the rate of ₹ 50/m² in the circular park. Then find the approximate cost of manuring

(a) ₹ 8338 (b) ₹ 833.8

(c) ₹ 4710 (d) ₹ 35325

Passage

Ram purchased a gift which is in the shape of a tetrahedron. Let A = (1, 2, 1), B = (2, 3, 1), C = (3, 1, 0) and D = (4, 3, 1) be the vertices of the tetrahedron.

Based on the above information, answer the following questions

46. The vector $\overrightarrow{AB}$ is

(a) $\hat{i}+\hat{j}$ (b) $\hat{i}+\hat{j}+\hat{k}$

(c) $\hat{i}-\hat{j}$ (d) $\hat{j}+\hat{k}$

47. The vector $\overrightarrow{CD}$ is

(a) $2\hat{i}+2\hat{j}+3\hat{k}$ (b) $\hat{i}+2\hat{j}+\hat{k}$

(c) $\hat{i}-\hat{j}+\hat{k}$ (d) $\hat{i}-2\hat{j}-\hat{k}$

48. The area of ΔABC is equal to

(a) $\frac{\sqrt{13}}{2}$ (b) $\frac{\sqrt{5}}{2}$

(c) $\frac{\sqrt{7}}{2}$ (d) $\frac{\sqrt{11}}{2}$

49. The unit vector along $\overrightarrow{CD}$ is

(a) $\frac{1}{\sqrt{6}}(\hat{i}+2\hat{j}+\hat{k})$ (b) $\frac{1}{\sqrt{3}}(\hat{i}-3\hat{j}-\hat{k})$

(c) $\frac{1}{\sqrt{6}}(\hat{i}-2\hat{j}+\hat{k})$ (d) $\frac{1}{\sqrt{2}}(\hat{i}-\hat{j})$

50. $\overrightarrow{AB}.\overrightarrow{CD}$ is

(a) −1 (b) 1 (c) −3 (d) 3

Hints & Explanations

1. (a) $\begin{bmatrix} 1-x^2 & 2y \\ x+5 & 0 \end{bmatrix} = \begin{bmatrix} -3 & y-1 \\ 7 & 0 \end{bmatrix}$

So, $x+5=7 \quad \Rightarrow x=2$

and $2y=y-1 \quad \Rightarrow y=-1$

2. (b) (A) If $A'=-A \Rightarrow A$ is skew symmetric

(B) If $A'=A \Rightarrow A$ is symmetric

(C) $(A^{-1})'=(A')^{-1}$

(D) $(AB)'=B'A'$ (Reversal law)

3. (d) Given,

$kx+3y+3z=5$
$x-2y+z=-4$
$3x-y-2z=3$

For no solution, $\begin{vmatrix} k & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{vmatrix}=0$

$\Rightarrow k(4+1)-3(-2-3)+3(-1+6)=0$

$\Rightarrow 5k+15+15=0 \Rightarrow k=-6$

and $D_1=\begin{vmatrix} 5 & 3 & 3 \\ -4 & -2 & 1 \\ 3 & -1 & -2 \end{vmatrix}$

$=5(4+1)-3(8-3)+3(4+6)$
$=25-15+30$

$D_1=40 \neq 0$

So, $k=-6$ for no solution

4. (b) Given, $e^y(x+1)=1$

$y+\ln(x+1)=\ln(1) \Rightarrow y=-\ln(x+1)$

Now, $\frac{dy}{dx}=\frac{-1}{x+1}$

$\frac{d^2y}{dx^2}=\frac{1}{(x+1)^2}=\left(\frac{1}{x+1}\right)^2$

$\Rightarrow \frac{d^2y}{dx^2}=\left(\frac{dy}{dx}\right)^2$. So, $k=2$

5. (b) Given the curve $y=e^{2x}$

Now, $\frac{dy}{dx}=2e^{2x}$

At (0, 2), $\frac{dy}{dx}=2\times1=2$

Since, equation of tangent line
$y-1=2(x-0) \Rightarrow y=2x+1$

At x-axis, $2x+1=0 \Rightarrow x=-\frac{1}{2}$

$\Rightarrow$ Required point $=\left(-\frac{1}{2},0\right)$

6. (a) Given, $f(x)=-2x^3-9x^2-12x+1$

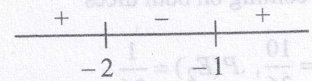
$f'(x)=-6x^2-18x-12$

For increasing of f, $f'(x)>0$

$\Rightarrow -6x^2-18x-12>0$

$\Rightarrow x^2+3x+2<0$

$\Rightarrow (x+2)(x+1)<0$



So, $x \in (-2,-1)$

7. (c) Since, $\int \frac{x}{\sqrt{x-1}}dx=f(x)$

Let $x-1=t^2 \Rightarrow dx=2t\,dt$

$\Rightarrow \int \frac{x}{\sqrt{x-1}}dx=\int \frac{t^2+1}{t}2t\,dt$

$=2\int(t^2+1)dt=2\left(\frac{t^3}{3}+t\right)+C$

$f(x)=\frac{2}{3}(x-1)^{3/2}+2\sqrt{x-1}+C$

So, $A=\frac{2}{3}, B=2$

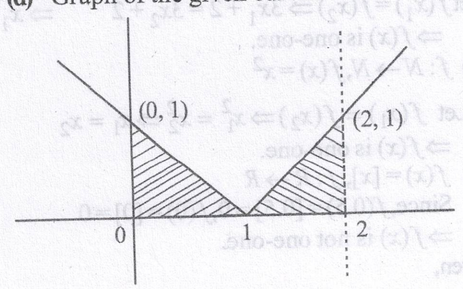
If $f(2)=0 \Rightarrow \frac{2}{3}\cdot1+2\cdot1+C=0 \Rightarrow C=-\frac{8}{3}$

If $f(1)=0 \Rightarrow 0+0+C=0 \Rightarrow C=0$

8. (a) $\int_0^5|x-5|dx=-\int_0^5(x-5)dx$

$=\left(\frac{x^2}{2}-5x\right)_5^0=-\left(\frac{25}{2}-25\right)=\frac{25}{2}$

9. (d) Graph of the given curves



Required Area $=\frac{1}{2}\times1\times1+\frac{1}{2}\times1\times1=\frac{1}{2}+\frac{1}{2}=1$

10. (c) (A) Degree = 2, (B) Degree = 3
(C) Degree = 1, (D) Degree = 4

11. (a) Given, $y = ae^{mx} + be^{nx}$
Now, $y' = ame^{mx} + bne^{nx}$
$y'' = am^2e^{mx} + bn^2e^{nx}$
Since, $y'' - my' - ny'$
$= am^2e^{mx} + bn^2e^{nx} - am^2e^{mx} - bmne^{nx} - amne^{mx} - bn^2e^{nx}$
$= -mn\,(ae^{mx} + be^{nx})$
$\Rightarrow\ y'' - (m+n)\,y' + mny = 0$

12. (b) Since, $E = \{(1, 6), (2, 6), (3, 6), (4, 6), (5,6), (6,6), (6,1), (6,2), (6,3), (6,4), (6,5)\}$
Let $E_1 = 6$ coming at exact ones
$E_2 = 6$ coming on both dices
So, $P(E_1) = \dfrac{10}{36}$, $P(E_2) = \dfrac{1}{36}$
So, expectations $= 1\times\dfrac{10}{36} + 2\times\dfrac{1}{36} = \dfrac{1}{3}$

13. (c) Given, $n = 10$
Since, $p + q = 1 \Rightarrow p = 1 - 0.7 = 0.3$
So, mean $= 10 \times 0.3 = 3$

14. (c) $C - B - A - E - D$

15. (d) Now, $Z_{(0,0)} = 0, Z_{(5,0)} = 2.5 = 10$
$Z_{(3,4)} = 6 + 4q,\ Z_{(0,5)} = 5q$
Since, $Z_{(3,4)} = Z_{(0,5)}$
$\Rightarrow\ 6 + 4q = 5q \Rightarrow q = 6$

16. (b) Given, $R = \{(1, 2), (2, 2), (1, 1), (1, 3), (4,4), (3,3), (3,2)\}$ and $A = \{1, 2, 3, 4\}$
$\forall\ x \in A, xRx \Rightarrow R$ is Reflexive Relation.
Since, $1R2 \not\Rightarrow 2R1 \Rightarrow R$ is not symmetric.
For any $a, b, c \in A$, $aRb, bRc \Rightarrow aRc$
$\Rightarrow\ R$ is transitive.

17. (b) Given, (A) $f(x) = x^2, f: R \to R$
For one-one, $f(x_1) = f(x_2)$ (Let)
$\Rightarrow\ x_1^2 = x_2^2 \Rightarrow\ x_1 = \pm x_2$
So $f(x) = x^2$ is not one-one.
(B) $f(x) = 3x + 2, f: R \to R$
Let $f(x_1) = f(x_2) \Rightarrow 3x_1 + 2 = 3x_2 + 2 \quad \Rightarrow x_1 = x_2$
$\Rightarrow f(x)$ is one-one.
(C) $f: N \to N, f(x) = x^2$
Let $f(x_1) = f(x_2) \Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = x_2$
$\Rightarrow f(x)$ is one-one.
(D) $f(x) = [x], f: R \to R$
Since, $f(0.5) = [0.5] = 0, f(0) = [0] = 0$
$\Rightarrow f(x)$ is not one-one.

18. (d) Given,
$$\tan^{-1}\left[2\cos\left(2\cos^{-1}\left(\frac{1}{2}\right)\right)\right] = \tan^{-1}\left[2\cos\left(2\times\frac{\pi}{6}\right)\right]$$
$$= \tan^{-1}\left[2\cos\left(\frac{\pi}{3}\right)\right] = \tan^{-1}\left(2\times\frac{1}{2}\right)$$
$$= \tan^{-1}(1) = \frac{\pi}{4}$$

19. (a) Given, $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$
$= x(-x^2 - 1) - \sin\theta(-x\sin\theta - \cos\theta) + \cos\theta\,(x\cos\theta - \sin\theta)$
$= -x^3 - x + x\sin^2\theta + \sin\theta\cos\theta + x\cos^2\theta - \sin\theta\cos\theta$
$= -x^9 - x + x\cdot 1 = -x^3$

20. (d) $\sin^{-1} x \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$
$\dfrac{-\pi}{2} \le \sin^{-1} x \le \dfrac{\pi}{2}$
So, other branch is
$2\pi - \dfrac{\pi}{2} \le \sin^{-1} x \le 2\pi + \dfrac{\pi}{2}$
$\dfrac{3\pi}{2} \le \sin^{-1} x \le \dfrac{5\pi}{2}$

21. (a) $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$
And, $A^2 = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$
$4A = 4\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 12 \\ 4 & 8 \end{bmatrix}$
Now, $A^2 - 4A + I$
$= \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ 4 & 8 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

22. (d) $f(x) = [x]$
Since, the greatest integer function is discontinuous at integers.

23. (b) $f: R \to R$ define as,
$$f(x) = \begin{cases} 2x - 5, & 2 < x < 3 \\ 2, & 3 \le x < 5 \\ 5x, & 5 \le x < 7 \end{cases}$$
At $x = 3$, $f(3^-) = 2\times 3 - 5 = 1$
$f(3^+) = 2$
Function discontinuous at $x = 3$
At $x = 5$,
$f(5^-) = 2$
$f(5^+) = 5x = 5 \times 5 = 25$
Function discontinuous at $x = 5$
So, 3, 5 is discontinuous points.

24. (a) If $f(x)=|x-a|$

then $f(x)$ is not differentiable at $x=a$

25. (a) $A=$ Assume the sides x, y

$B=$ Write perimeter and area

$C=$ Equate area to zero

$D=$ Derivate the area

$E=$ Check $\frac{d^2A}{dx^2}<0$ at critical points.

26. (d) Let $I=\int e^{2x}\sin x\,dx$

$$I=\left(\int e^{2x}\,dx\right)\sin x-\int\left(\cos x\cdot\int e^{2x}dx\right)dx$$

$$I=\sin x\frac{e^{2x}}{2}-\left[\frac{e^{2x}}{4}\cos x+\int\left(\sin x\frac{e^{2x}}{4}\right)dx\right]$$

$$I=\sin x\frac{e^{2x}}{2}-\frac{e^{2x}}{4}\cos x-\frac{I}{4}$$

$$\Rightarrow \frac{5I}{4}=e^{2x}\left(\frac{\sin x}{2}-\frac{\cos x}{4}\right)+C$$

$$\Rightarrow I=\frac{e^{2x}}{5}(2\sin x-\cos x)+C$$

27. (d) By using formula of integration

$A\rightarrow IV, B\rightarrow III, C\rightarrow II, D\rightarrow I$

(d) is correct answer.

28. (c) Let $I=\int_0^{\pi/2}\frac{dx}{1+\tan^n x}$, put $\tan x=\frac{\sin x}{\cos x}$

$$I=\int_0^{\pi/2}\frac{\cos^n x}{\sin^n x+\cos^n x}dx \quad \text{.... (i)}$$

By definite integral properly

$$I=\int_0^{\pi/2}\frac{\cos^n\left(\frac{\pi}{2}-x\right)}{\sin^n\left(\frac{\pi}{2}-x\right)+\cos^n\left(\frac{\pi}{2}-x\right)}$$

$$\Rightarrow I=\int_0^{\pi/2}\frac{\sin^n x}{\cos^n x+\sin^n x}dx \quad \text{.... (ii)}$$

Add equations (i) and (ii),

$$2I=\int_0^{\pi/2}\frac{\cos^n x+\sin^n x}{\cos^n x+\sin^n x}dx=\int_0^{\pi/2}1\,dx$$

$$2I=[x]_0^{\pi/2}=\frac{\pi}{2}\Rightarrow I=\frac{\pi}{4}$$

29. (d) (A) Let

$$I=\int_0^{\pi/2}\sin^5 x\,dx=\int_0^{\pi/2}(1-\cos^2 x)^2\sin x\,dx$$

Put $\cos x=t\Rightarrow -\sin x\,dx=dt$

$$\Rightarrow I=-\int_1^0(1-t^2)^2\,dt=\frac{1}{5}-\frac{2}{3}+1=\frac{8}{15}$$

(B) Let $I=\int_0^{\pi/2}\log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$

$$=\int_0^{\pi/2}\log\left(\frac{4+3\sin\left(\frac{\pi}{2}-x\right)}{4+3\cos\left(\frac{\pi}{2}-x\right)}\right)dx$$

$$\Rightarrow I=\int_0^{\pi/2}\log\left(\frac{4+3\cos x}{4+3\sin x}\right)dx$$

$$2I=\int_0^{\pi/2}\log\left(\frac{4+3\sin x}{4+3\cos x}\right)\left(\frac{4+3\cos x}{4+3\sin x}\right)dx$$

$\Rightarrow 2I=0 \Rightarrow I=0$

(C) Let $I=\int_{\pi/6}^{\pi/3}\frac{1}{1+\sqrt{\tan x}}dx$

$$=\int_{\pi/6}^{\pi/3}\frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$

$$=\int_{\pi/6}^{\pi/3}\frac{\sqrt{\cos\left(\frac{\pi}{6}+\frac{\pi}{3}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{6}+\frac{\pi}{3}-x\right)}+\sqrt{\cos\left(\frac{\pi}{6}+\frac{\pi}{3}-x\right)}}dx$$

$$\Rightarrow I=\int_{\pi/6}^{\pi/3}\frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$$

$$\Rightarrow 2I=\int_{\pi/6}^{\pi/3}\frac{\left(\sqrt{\cos x}+\sqrt{\sin x}\right)}{\sqrt{\cos x}+\sqrt{\sin x}}dx=\frac{\pi}{3}-\frac{\pi}{6}$$

$$\Rightarrow I=\frac{1}{2}\left[\frac{\pi}{3}-\frac{\pi}{6}\right]=\frac{\pi}{12}$$

(D) Let $I=\int_0^1 x e^x\,dx$

$$\Rightarrow I=xe^x-\int_0^1 e^x\,dx$$

$$\Rightarrow I=\left[xe^x-e^x\right]_0^1=(e^1-e^1)-(0-1)=1$$

So, $D>A>C>B$

30. **(a)** $\int \frac{\sin(x+\alpha)}{\sin x}dx = \int\left(\frac{\sin x\cos\alpha + \cos x\sin\alpha}{\sin x}\right)dx$

$= \int \cos\alpha\, dx + \int \cot x \sin\alpha\, dx$

$= x\cos\alpha + \sin\alpha \log(\sin x) + C$

31. **(b)** (A) Let, I = $\int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x + \cos^4 x}dx$

$= \int_0^{\pi/2} \frac{\sin^4\left(\frac{\pi}{2}-x\right)}{\sin^4\left(\frac{\pi}{2}-x\right)+\cos^4\left(\frac{\pi}{2}-x\right)}dx$

$\Rightarrow I = \int_0^{\pi/2} \frac{\cos^4 x}{\sin^4 x + \cos^4 x}dx$

Now,

$2I = \int_0^{\pi/2} \frac{(\cos^4 x + \sin^4 x)}{\cos^4 x + \sin^4 x}dx = \int_0^{\pi/2} dx = \frac{\pi}{2}$

$\Rightarrow I = \frac{\pi}{4}$

(B) Let $f(x) = \sin^5 x \cos^4 x$

$f(-x) = -\sin^5 x \cos^4 x = -f(x)$

So, $\int_{-1}^{1} \sin^5 x \cos^4 x = 0$

(C) $\int_1^{\sqrt{3}} \frac{dx}{1+x^2} = \left[\tan^{-1} x\right]_1^{\sqrt{3}} = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$

(D) $\int_2^3 x^2 dx = \left[\frac{x^3}{3}\right]_2^3 = \frac{27}{3} - \frac{8}{3} = \frac{19}{3}$

$D > A > C > B$

32. **(c)** Let $I = \int_0^a \frac{\cos x}{1+\sin^2 x}dx = \frac{\pi}{4}$

$\sin x = t \Rightarrow \cos x\, dx = dt$

$\int \frac{dt}{1+t^2} = \tan^{-1} t$

$I = \left[\tan^{-1}(\sin x)\right]_0^a = \tan^{-1}(\sin a) - 0$

$\tan^{-1}(\sin a) = \frac{\pi}{4} \Rightarrow \sin(a) = 1 \Rightarrow a = \frac{\pi}{2}$

33. **(d)** $\frac{dy}{dx} = \frac{x^2+y^2}{xy}$

Let $y = Vx \Rightarrow \frac{dy}{dx} = V + x\frac{dV}{dx}$

$V + x\frac{dv}{dx} = \frac{x^2 + V^2x^2}{Vx^2}$

$\Rightarrow V + x\frac{dV}{dx} = \frac{1+V^2}{V} \Rightarrow x\frac{dV}{dx} = \frac{1}{V}$

$\Rightarrow VdV = \frac{dx}{x}$

$\Rightarrow \frac{V^2}{2} = \log x + \log c \Rightarrow \frac{y^2}{2x^2} = \log cx$

34. (a) (A) distance of (3, 4, 5) from (0, 4, 0) is

$d = \sqrt{(3-0)^2 + 0 + (5-0)^2} = \sqrt{34}$

(B) $B \rightarrow$ III

(C) Projection $= \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = \frac{2}{\sqrt{6}}$

(D) $\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{1}{3} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{3}\right)$

35. **(c)** Given the plane, $x + 2y - 2z + 9 = 0$

Now, required distance

$= \frac{|3\cdot1 + (-2)\cdot2 + 1\cdot(-2) + 9|}{\sqrt{1^2 + 2^2 + (-2)^2}}$

$= \frac{|3-4-2+9|}{\sqrt{1+4+4}} = \frac{6}{3} = 2$ unit

36. **(b)** Given the plane $x + y - z + 2 = 0$

(A) The plane $x + y - z + 5 = 0$

So, required distance

$= \frac{|5-2|}{\sqrt{1^2+1^2+1^2}} = \frac{3}{\sqrt{3}} = \sqrt{3}$ unit

(B) The plane $x + y - z + \frac{5}{2} = 0$

So, required distance

$= \frac{\left|\frac{5}{2}-2\right|}{\sqrt{1+1+1}} = \frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{6}$ unit

(C) The plane is $x + y - z + 3 = 0$

So, required distance

$= \frac{|3-2|}{\sqrt{1+1+1}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ unit

(D) The plane is $x + y - z + 2 = 0$

So, required distance

$= \frac{|2-2|}{\sqrt{1+1+1}} = 0$ unit So D, B, C, A.

37. **(bonus),** Let $\frac{x-4}{2} = \frac{y}{3} = \frac{z+2}{4} = t$

$\Rightarrow x = 2t+4, y = 3t, z = 4t-2$

Now, $2(2t+4) - 2(3t) + 4t - 2 = 0$

$\Rightarrow 4t+8-6t+4t-2=0$

$\Rightarrow 2t+6=0 \Rightarrow t=-3$

So, the point is $(-2,-9,-14)$.

Now, required distance

$=\sqrt{(-2)^2+(-9)^2+(-14)^2}$

$=\sqrt{4+81+196}=\sqrt{281}$ unit

38. (b) Given, $P(A)=\frac{1}{4},\ P(B)=\frac{1}{2}$

Now,

$$P\left(\frac{A'}{B'}\right)=\frac{P(A'\cap B')}{P(B')}=\frac{1-P(A\cup B)}{1-P(B)}$$

$$=\frac{1-(P(A)+P(B)-P(A\cap B))}{1-\frac{1}{2}}$$

$$=2\left(1-\frac{1}{4}-\frac{1}{2}+\frac{1}{4}\times\frac{1}{2}\right)=\frac{3}{4}$$

39. (c) (A) We know, $P(x=0\le x\le 4)=1$

$\Rightarrow k+\frac{1}{4}+6k+4k+k=1 \Rightarrow 12k=\frac{3}{4}$

$\Rightarrow k=\frac{1}{16}$

(B) $P(x=3)=4\times\frac{1}{16}=\frac{1}{4}$

(C) $P(x<2)=k+\frac{1}{4}=\frac{1}{16}+\frac{1}{4}=\frac{5}{16}$

(D) $P(1<x<4)=6k+4k=10k=\frac{10}{16}=\frac{5}{8}$

(E) $P(x=4)=k=\frac{1}{16}$

40. (d) Given, $z=3x+9y$

Now, $Z_{(0,10)}=0+9\times 10=90$

$Z_{(5,5)}=3.5+9.5=60$

$Z_{(15,15)}=3.15+9.15=180$

$Z_{(0,20)}=0+9.20=180$

41. (c) Equation of circle, $x^2+y^2=(15)^2$

$x=15\sin\theta,\ dx=15\cos\theta\, d\theta$

$A=4\int_0^r\sqrt{r^2-x^2}\,dx$ where $r=15$

$A=4\int_0^{15}\sqrt{225-x^2}\,dx$

42. (c) By integral formula

$$\int\sqrt{a^2-x^2}\,dx=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}$$

then, $\int\sqrt{225-x^2}\,dx=\frac{x}{2}\sqrt{225-x^2}+\frac{225}{2}\sin^{-1}\frac{x}{15}$

43. (d) Area of pavement = Total area – area of park

$$A=4\int_0^{17}\sqrt{289-x^2}\,dx-4\int_0^{15}\sqrt{225-x^2}\,dx$$

44. (d) $A=4\int_0^{17}\sqrt{289-x^2}\,dx-4\int_0^{15}\sqrt{225-x^2}\,dx$

$A=289\pi-225\pi=64\pi$ sq. units

45. (d) Area of park $=225\pi$ sq. units

Total cost is $=225\pi\times 50=35325$

46. (a) $\vec{A}=(1,\ 2,\ 1),\ \vec{B}=(2,\ 3,\ 1)$

$\overrightarrow{AB}=\vec{B}-\vec{A}=(1,\ 1,\ 0)=\hat{i}+\hat{j}$

47. (b) $\vec{C}=(3,\ 1,\ 0),\ \vec{D}=(4,\ 3,\ 1)$

$\overrightarrow{CD}=\vec{D}-\vec{C}=(1,\ 2,\ 1)=\hat{i}+2\hat{j}+\hat{k}$

48. (d) $\overrightarrow{AB}=\hat{i}+\hat{j},\ \overrightarrow{BC}=\hat{i}-2\hat{j}-\hat{k}$

Area $=\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{BC}|$

$$=\frac{1}{2}\left|\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 0\\ 1 & -2 & -1\end{vmatrix}\right|=\frac{|(-\hat{j}+\hat{i}-3\hat{k})|}{2}=\frac{\sqrt{11}}{2}$$

49. (a) $\overrightarrow{CD}=\hat{i}+2\hat{j}+\hat{k} \Rightarrow |\overrightarrow{CD}|=\frac{\hat{i}+2\hat{j}+\hat{k}}{\sqrt{6}}$

50. (d) $\overrightarrow{AB}=\hat{i}+\hat{j},\ \overrightarrow{CD}=\hat{i}+2\hat{j}+\hat{k}$

$\Rightarrow \overrightarrow{AB}\cdot\overrightarrow{CD}=(1+2)=3$